

Three modes of the shear-flow-driven ion cyclotron instability

D. V. Chibisov, V.S. Mikhailenko and K.N. Stepanov

Kharkov National University, 61108 Kharkov, Ukraine

Abstract

The electrostatic shear-flow-driven ion cyclotron instability of magnetic field aligned sheared plasma flow is investigated analytically. It is shown that the shear-flow-driven electrostatic ion cyclotron instability can be considered as a combination of three different instabilities determined by their own mechanism of excitation: ion-kinetic, ion-hydrodynamic and electron-kinetic. Each of these instabilities are dominant in different ranges of the wavelength along the magnetic field. The linearized dispersion equation is solved within the limits of long and short waves along the magnetic field where effects of electrons and ions, respectively, are dominant in the development of these instabilities. The general criterion of instability excitation, which couples the flow velocity shear and wave number across the magnetic field, is obtained.

1 INTRODUCTION

The ion cyclotron instabilities of magnetic field-aligned sheared plasma flows have been intensively investigated for the last two decades [1-11]. Interest to this problem was caused by the attempts to explain the anomalously strong ion heating in a number of the satellites' observations where the correlation in place and time of the broadband, low-frequency waves, and transverse ion energization was detected at the boundaries of plasma structures [12-16]. As it turned out, the shear flow of ions along the magnetic field, together with the magnetic field-aligned electron current and others mechanisms is a possible source of free energy for the development of EIC instabilities. It was suggested in Ref. [1] that the ion cyclotron waves in the magnetopause are excited by the magnetic-field aligned ion flows with a transverse velocity gradient. In Ref.[1], a dispersion equation for electrostatic ion cyclotron (EIC) waves in plasma shear flows has been obtained and solved numerically in the long wavelength limit along the magnetic field. It was shown that these oscillations have short wavelength in the transverse to the magnetic field direction. Ganguly *et al.* [2-4] have proved that ion cyclotron instability may be developed due to the combined effect of the inverse ion cyclotron damping and velocity shear of the magnetic field-aligned ion flow. It was obtained in Ref. [2-4] that different cyclotron harmonics have approximately the same excitation conditions, which explains the presence in the ion cyclotron waves spectrum in the Earth's auroral magnetosphere numbers of cyclotron harmonics. By using a hydrodynamic treatment, Merlino [5] has shown the possibility of excitation of ion cyclotron instability due to shear of the ion flow. Mikhailenko *et al.*[6, 7] using the kinetic approach, have

found that the shear flow of ions could lead to the splitting of EIC oscillations onto two modes. One of them is the shear-modified EIC mode, which also exists in plasma flows without velocity shear and can be unstable due to the field-aligned electron current. The second mode is caused by the shear flow of ions and can be unstable even in a current-free plasma due to combined effect of the velocity shear and inverse of electron Landau damping. It was obtained in Refs.[6, 7], that EIC waves may be excited by the hydrodynamic mechanism, if the value of velocity shear flow and wave numbers satisfy the certain conditions. It was shown in Refs.[5, 6, 7], that velocity shear-induced ion cyclotron instability in the limit of long wavelength along the magnetic field becomes similar to an aperiodic instability considered by D'Angelo[17] and is, in fact, the extension of this instability onto the ion cyclotron frequency range. The excitation of shear-flow-driven EIC instability in a collisional plasma, which is the signature of the bottomside ionosphere was studied by Mikhailenko *et al.*[8]. That instability is excited due to electron-neutral collisions in the limit of long wave length of the unstable waves along the magnetic field, for which effect of the electron inverse Landau damping is negligible. Kaneko *et al.* [11] have carried out the three-dimensional electrostatic particle simulations and shown the possibility of shear-flow-driven EIC instability. Simultaneously, a number of studies (see for example Refs.[9, 10]) attempted to detect in the Q - machine the shear-flow-driven EIC instability. However, as stated in Ref.[10] this instability was not observed yet.

In this paper, we analyze the excitation mechanisms of the shear-flow-driven EIC instabilities in a collisionless currentfree sheared plasma. We find that shear-flow-driven ion cyclotron instability is developed by different mechanisms of excitations, which dominate in certain ranges of the wavelength along the magnetic field. According to that we find reasonable to define three different modes of that instability: ion-kinetic mode, ion-hydrodynamic mode and electron-kinetic mode. The regions of the of the wavelength along the magnetic field, where these modes are dominate and corresponding growth rates are found here. We have obtained also the criterion of shear-flow-driven EIC instabilities for the different ion cyclotron harmonics, which connects the flow velocity shear and wave numbers across the magnetic field.

The paper is organized as follows. Sec. 2 is devoted to the analysis of the shear-flow-driven EIC instabilities of the fundamental ion cyclotron harmonic. In Sec. 3 the properties of these instabilities for an arbitrary high number of the ion cyclotron harmonic are considered. Conclusions are given in Sec. 4.

2 THE INSTABILITY OF THE FIRST CYCLOTRON HARMONIC

We start with dispersion equation for the oscillations in the ion cyclotron frequency range in homogeneous magnetic-field aligned plasma shear flow, which is given by [3, 6]

$$\begin{aligned} \varepsilon(\mathbf{k}, \omega) = & 1 + \frac{1}{k^2 \lambda_{De}^2} (1 + i\sqrt{\pi} z_{e0} (z_{e0})) + \frac{1}{k^2 \lambda_{Di}^2} \left[1 - \frac{k_y}{k_z} S_i + i\sqrt{\pi} \sum_{n=-\infty}^{\infty} W(z_{in}) A_n(b_i) \right. \\ & \left. \times \left(\frac{\omega - k_z V_{0i}}{\sqrt{2} k_z v_{Ti}} - \frac{k_y}{k_z} S_i z_{in} \right) \right] = 0, \end{aligned} \quad (1)$$

where $\lambda_{D\alpha}$ is the Debye length, $A_n(b_i) = I_n(b_i) e^{-b_i}$, I_n is the modified Bessel function, $b_i = k_\perp^2 \rho_{Ti}^2$, $\rho_{Ti} = v_{Ti}/\omega_{ci}$ is the thermal Larmor radius, $S_i = dV_{0i}(X)/\omega_{ci} dX$ is the normalized flow velocity shear, $z_{\alpha n} = (\omega - n\omega_{c\alpha} - k_z V_{0\alpha})/\sqrt{2} k_z v_{T\alpha}$ and $W(z) = e^{-z^2} \left(1 + (2i/\sqrt{\pi}) \int_0^z e^{t^2} dt \right)$.

We consider first the instability of the fundamental cyclotron harmonic, assuming that $\omega(\mathbf{k}) = \omega_{ci} + k_z V_{0i} + \delta\omega(\mathbf{k})$ with $\delta\omega(\mathbf{k}) \ll \omega_{ci}$. Assume that the z_{in} argument of the W - function in the sum over cyclotron harmonics has an arbitrary value for the fundamental harmonic, while $|z_{in}| > 1$ in the remaining sum. That is valid when the inequality $k_z \rho_{Ti} < 1$ is satisfied. Using the asymptotic form for W - function for large argument values, $W(z_i) \sim e^{-z_i^2} + (i/\sqrt{\pi} z_i) (1 + 1/2z_i^2)$, we carry out the summation over the cyclotron harmonics for $n \neq 1$

$$\sum_{n \neq 1} W(z_{in}) A_n(b_i) \left(\frac{\omega - k_z V_{0i}}{\sqrt{2} k_z v_{Ti}} - \frac{k_y}{k_z} S_i z_{in} \right) \approx -\frac{1 - A_0(b_i)}{b_i} - \frac{k_y}{k_z} S_i (1 - A_0(b_i)). \quad (2)$$

Then dispersion relation (1) reduces to the form

$$\begin{aligned} k^2 \lambda_{Di}^2 \varepsilon(\mathbf{k}, \omega) = & 1 + \tau (1 + i\sqrt{\pi} z_{e0} W(z_{e0})) - \frac{1 - A_0(b_i)}{b_i} - \frac{k_y}{k_z} S_i A_1(b_i) \\ & + i\sqrt{\pi} W(z_{i1}) A_1(b_i) \left(\frac{\omega_{ci}}{\sqrt{2} k_z v_{Ti}} + z_{i1} - \frac{k_y}{k_z} S_i z_{i1} \right) = 0. \end{aligned} \quad (3)$$

We use in what follows the normalized wavelength along the magnetic field, $\lambda = 1/k_z \rho_{Ti}$, instead of variable k_z . Considering z_{i1} as the normalized complex frequency, we find the solution $z_{i1}(\lambda)$ of the Eq. (3) for EIC instability in the short wavelength limit, at which the electrons are adiabatic with $z_{e0} \ll 1$ and instability is developed due to the inverse of ion cyclotron damping, and in long wavelength limit, at which ion cyclotron damping is negligible and EIC instability is developed due to inverse electron Landau damping.

We find first the short wavelength threshold. It is important to note, that in shearless plasma flow the ion cyclotron instability does not developed in this limit. The threshold values for variables λ and z_{i1} we obtain by equating to zero the real and imaginary parts of Eq. (3),

$$\begin{cases} \lambda/\sqrt{2} + z_{i1} - \lambda k_y \rho_{Ti} S_i z_{i1} = 0, \\ 1 + \tau - (1 - A_0(b_i))/b_i - \lambda k_y \rho_{Ti} S_i A_1(b_i) = 0. \end{cases} \quad (4)$$

This set has a solution when inequality $k_y \rho_{Ti} S_i > 0$ is met. For this case we obtain the short-wavelength threshold value λ_{1s} for the excitation of the instability, as well as the threshold value of the normalized complex frequency z_{1s} , which is the real at that threshold,

$$\lambda_{1s} = \frac{1}{k_y \rho_{Ti} S_i A_1(b_i)} \left(1 + \tau - \frac{1 - A_0(b_i)}{b_i} \right), \quad (5)$$

$$z_{1s} = \frac{1}{\sqrt{2} S_i k_y \rho_{Ti}} \left(1 + \frac{A_1(b_i)}{1 - G_1 + \tau} \right), \quad (6)$$

where $G_1 = A_1(b_i) + (1 - A_0(b_i))/b_i$ and index s means the short-wavelength instability threshold of the first cyclotron harmonic. The value $\delta\omega = \delta\omega_{01}$ at the instability threshold is

$$\delta\omega_{01} = \frac{\omega_{ci} A_1(b_i)}{1 - G_1 + \tau}, \quad (7)$$

which coincides with value of $\delta\omega_{01}$ in shearless flow. The approximate solution to Eq.(3) for z_{i1} at the vicinity of instability threshold we obtain by Taylor series expansion of Eq.(3) in powers of $(\lambda - \lambda_{1s})$, with zero-order and linear terms retained,

$$z_{i1} \simeq z_{01} + z'_\lambda(\lambda_{1s})(\lambda - \lambda_{1s}). \quad (8)$$

Here $z'_\lambda(\lambda_{1s}) = -\varepsilon'_\lambda/\varepsilon'_z$ with

$$k^2 \lambda_{Di}^2 \varepsilon'_\lambda(z_{1s}) = -i\sqrt{\pi} W(z_{1s}) A_1(b_i) \frac{z_{1s}}{\lambda_{1s}} - k_y \rho_{Ti} S_i A_1 \quad (9)$$

and

$$k^2 \lambda_{Di}^2 \varepsilon'_z(z_{1s}) = -i\sqrt{\pi} W(z_{1s}) A_1(b_i) \frac{\lambda_{1s}}{\sqrt{2} z_{1s}}. \quad (10)$$

The dispersive part, $\delta\omega$, of the ion cyclotron wave frequency and the growth rate at the vicinity of the instability threshold can be obtained from Eq. (6) as

$$\delta\omega \simeq \delta\omega_{01} \frac{\lambda_{1s}}{\lambda} + \delta\omega_{01} \left(\frac{\sqrt{2} k_y \rho_{Ti} S_i \text{Im} W(z_{1s})}{\sqrt{\pi} |W(z_{1s})|^2} - \frac{A_1(b_i)}{1 - G_1 + \tau} \right) \left(1 - \frac{\lambda_{1s}}{\lambda} \right), \quad (11)$$

$$\gamma \simeq \delta\omega_{01} \frac{\sqrt{2} k_y \rho_{Ti} S_i \text{Re} W(z_{1s})}{\sqrt{\pi} |W(z_{1s})|^2} \left(1 - \frac{\lambda_{1s}}{\lambda} \right). \quad (12)$$

Because $\gamma \propto \text{Re} W(z_{1s}) = \exp(-z_{1s}^2)$, the instability growth rate is exponentially small at the vicinity of the threshold for the values of velocity shear flow and transverse wave number such that $\sqrt{2} k_y \rho_{Ti} S_i < 1$ and it is not exponentially small with opposite inequality $\sqrt{2} k_y \rho_{Ti} S_i > 1$ i. e. for larger values of the velocity shear and transverse wave numbers. The magnitude of the growth rate is affected also by the factor $k_y \rho_{Ti} A_1(b_i)$; the growth rate decreases as $(k_y \rho_{Ti})^3$ for long waves with $k_y \rho_{Ti} < 1$, whereas the growth rate varies slightly for the waves with $k_y \rho_{Ti} \gtrsim 1$.

Thus, the EIC instability at the vicinity of the short wave threshold is resulted from the combined effect of the inverse of ion cyclotron damping velocity shear and is the ion-kinetic mode

of the shear-flow-driven EIC instability. The most unstable waves are those with transverse wave numbers $\sqrt{2}k_y\rho_{Ti}S_i > 1$ and $k_y\rho_{Ti} \gtrsim 1$. In addition, since the development of this mode is caused by the thermal motion of ions along the magnetic field, this instability will be resulted in the longitudinal heating of ions.

The growth rate of the ion-kinetic mode of the shear-flow-driven EIC instability increases with an increase of wavelength along the magnetic field. Simultaneously, the value $|z_{i1}|$ also increases. If the transverse wave numbers meet the inequality $\sqrt{2}k_y\rho_{Ti}S_i < 1$, the growth rate, remaining exponentially small quantity, reaches a maximum at a certain value λ and then rapidly decreases. If the transverse wave numbers meet the inequality $\sqrt{2}k_y\rho_{Ti}S_i > 1$, the ion-kinetic mode with an increase $|z_{i1}| > 1$ turn into the ion-hydrodynamic mode. The dispersion equation for the ion-hydrodynamic mode of EIC shear-flow-driven instability can be obtained from Eq. (3) by using the asymptotic form of W - function for a large magnitudes of $|z_{i1}| > 1$

$$1 + \tau (1 + i\sqrt{\pi}z_{e0}W(z_{e0})) - G_1 - \frac{A_1(b_i)}{\sqrt{2}z_{i1}}\lambda + \frac{A_1(b_i)}{2z_{i1}^2}k_y\rho_{Ti}S_i\lambda = 0, \quad (13)$$

where $z_{e0} = \lambda/\sqrt{2\mu}$ and $\mu = m_i/m_e$. The solution of Eq.(12) is

$$z_{i1} = \left[\lambda A_1(b_i) r \pm \sqrt{(\lambda A_1(b_i))^2 - 4(1 + \tau(1 + i\sqrt{\pi}z_{e0}W(z_{e0})) - G_1) S_i \lambda k_y \rho_{Ti} A_1(b_i)} \right] \times \left[2\sqrt{2}k_y\rho_{Ti}(1 + \tau(1 + i\sqrt{\pi}z_{e0}W(z_{e0})) - G_1) \right]^{-1}. \quad (14)$$

The long wavelength threshold for ion-hydrodynamic mode we find from Eq.(10) assuming $\text{Re}(W(z_{e0})) = 0$. This mode is developed when the wavelength λ meets the inequality

$$\lambda < \lambda_{1lh} = \frac{4k_y\rho_{Ti}S_i}{A_1(b_i)}(1 + \tau - G_1), \quad (15)$$

where index $1lh$ means the long-wavelength threshold of the ion-hydrodynamic mode of the first cyclotron harmonic. The dispersion of the ion cyclotron waves and the growth rate obtained from Eq. (14) are:

$$\delta\omega = \frac{\delta\omega_{01}}{2}, \quad \gamma = \frac{\delta\omega_{01}}{2} \sqrt{\frac{\lambda_{1lh} - \lambda}{\lambda}}. \quad (16)$$

Note, that $\delta\omega$ does not depend on the variable λ . The growth rate of the ion-hydrodynamic mode away from the threshold of stability, as well as for the ion-kinetic mode, varies slightly with changes of $k_y\rho_{Ti}$ for $k_y\rho_{Ti} \gtrsim 1$, while for $k_y\rho_{Ti} < 1$ the growth rate decreases as $(k_y\rho_{Ti})^{3/2}$. The development of hydrodynamic mode of the shear-flow-driven EIC instability will manifest itself in the turbulent heating of ions across the magnetic field due to the effect of ion cyclotron resonances broadening.

With a decrease of λ the growth rate increases until the value $|z_{i1}|$ becomes of order unity, where asymptotic form of W - function at $|z_{i1}| > 1$ is non-applicable. As it was shown by numerical estimates, the maximum growth rate occurs at $|z_{i1}| \simeq 1$, i. e. on the boundary of the ion-kinetic and ion-hydrodynamic modes of this instability.

When the inequality $\lambda > \lambda_{1lh}$ holds, the ion cyclotron oscillations split in two hydrodynamically stable modes for which $\delta\omega$ is equal to

$$\delta\omega = \frac{\delta\omega_{01}}{2} \left(1 \pm \sqrt{\frac{\lambda - \lambda_{1lh}}{\lambda}} \right). \quad (17)$$

The dispersion with sign plus corresponds to the EIC mode modified by the flow velocity shear and exists in a plasma without shear flow. Second mode with sign minus exists only in a plasma with a shear flow at the same condition as ion-kinetic and ion-hydrodynamic modes, namely at $\sqrt{2}k_y\rho_{Ti}S_i > 1$. It is the shear-flow-driven electron-kinetic EIC mode. For the case of a plasma with no electron current only shear-flow-driven EIC mode may be unstable due to combined effect of an inverse electron Landau damping and velocity shear. The growth rate of this electron-kinetic EIC mode is approximately [6]

$$\gamma \approx \omega_{ci} \frac{\sqrt{\pi}\tau k_y^2 \rho_{Ti}^2 S_i^2}{A_1(b_i) \sqrt{2\mu\lambda(\lambda - \lambda_{1lh})}} e^{-\lambda^2/2\mu}. \quad (18)$$

The long-wavelength limit of the electron-kinetic mode may be evaluated from the condition $z_{e0} = \lambda/\sqrt{2\mu} < 1$, which gives the threshold value $\lambda_{1lh} < \sqrt{2\mu}$ which define the additional condition for the excitation of the electron-kinetic mode. The estimates performed by Amatucci[12] for ion flows propagating in the Earth's ionosphere give the normalized flow velocity shear $S_i \sim 0.4$ for hydrogen ions and $S_i \sim 6$ for oxygen ions. The conditions of the shear-flow-driven EIC instability excitation are satisfied for hydrogen ions at $k_y\rho_{Ti} \gtrsim 3$ and for oxygen ions at $k_y\rho_{Ti} \gtrsim 1$. Then the inequality $\lambda_{1lh} < \sqrt{2\mu}$ for oxygen ions is met, whereas for hydrogen ions it is not satisfied. Thus for the conditions in the Earth's ionosphere the electron-kinetic mode of the shear-flow-driven EIC instability may be excited only for the flows of heavy (oxygen) ions. In addition, the development of electron kinetic mode leads to heating of the electrons along the magnetic field as well as to the turbulent heating of ions across the magnetic field.

We numerically solved the dispersion equation (1) and obtained the plots for the dispersion and growth rate depending on the normalized wavelength for oxygen ions (Fig. 1). The growth rate was calculated for two cases: with neglecting electron Landau damping (dashed line) and with accounting of it (dashed-dotted line). The first curve presents the ion-kinetic and ion-hydrodynamic modes of the shear-flow-driven EIC instability. One can see that the limiting wavelengths are equal approximately to 4 and 76 units; these values are close to theoretical magnitudes, which were calculated from Eqs. (5) and (15). The growth rate has maximal value for λ values for which $|z_{1i}| \simeq 1$. Electron Landau damping leads to a decrease of the growth rate of the ion-hydrodynamic mode, however the waves with the wavelengths $\lambda > \lambda_{1lh}$ become unstable (second curve). In this case, we have the electron-kinetic mode of the shear-flow-driven EIC instability. As it is seen from Fig. 1, the EIC oscillations split into two modes at $\lambda > \lambda_{1lh}$, that is predicted by Eq. (15).

So, the first harmonic of the shear flow-driven EIC instability can be considered as a set of three modes, the ion-kinetic mode, the ion-hydrodynamic mode and the electron-kinetic mode. Each

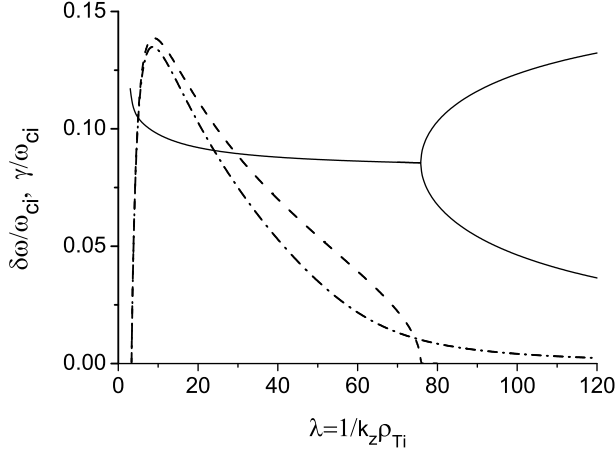


Figure 1: The dispersion (solid line) and the growth rate (dashed and dashed-dotted lines) of the EIC oscillations versus the normalized wavelength. Dashed line - the growth rate with neglecting the Landau damping by electrons, dashed-dotted line - the growth rate accounted for the Landau damping by electrons. $S_i = 3$, $k_\perp \rho_{Ti} = 1$.

of them exists in a certain ranges of wavelengths along the magnetic field, however they have common signatures such as condition $\sqrt{2}k_y \rho_{Ti} S_i > 1$ for their excitation and the growth of the growth rate with an increase of $k_y \rho_{Ti}$.

3 THE INSTABILITY OF THE HIGH CYCLOTRON HARMONIC

Now we investigate the instability of high cyclotron harmonic, $\omega(\mathbf{k}) = n'\omega_{ci} + k_z V_{0i} + \delta\omega(\mathbf{k})$ with $|n'| \geq 2$ and $\delta\omega(\mathbf{k}) \ll \omega_{ci}$. Assume that z_{in} in the sum over cyclotron harmonics has an arbitrary value for the $n = n'$ term, while in remaining sum $|z_{in}| > 1$, for which the asymptotic form of W - function for large argument values may be used. The summation over cyclotron harmonics at $k_y \rho_{Ti} \gg 1$ gives approximately

$$\sum_{n \neq n'} W(z_{in}) A_n(b_i) \left(\frac{\omega - k_z V_{0i}}{\sqrt{2}k_z v_{Ti}} - \frac{k_y}{k_z} S_i z_{in} \right) \approx \psi(z_\perp) - \frac{k_y}{k_z} S_i (1 - A_{n'}(b_i)), \quad (19)$$

where $z_\perp = (n'\omega_{ci} + \delta\omega)/\sqrt{2}k_y v_{Ti} \approx n'/k_y \rho_{Ti}$ and $\psi(z_\perp) = -2z_\perp e^{-z_\perp^2} \int_0^{z_\perp} e^{t^2} dt$. Then dispersion equation (3) can be presented in the form

$$\begin{aligned} k^2 \lambda_{Di}^2 \varepsilon(\mathbf{k}, \omega) &= 1 + \tau \left(1 + i\sqrt{\pi} z_{e0} W(z_{e0}) \right) + \psi(z_\perp) - \frac{k_y}{k_z} S_i A_{n'}(b_i) \\ &+ i\sqrt{\pi} W(z_{in'}) A_{n'}(b_i) \left(\frac{n'\omega_{ci}}{\sqrt{2}k_z v_{Ti}} + z_{in'} - \frac{k_y}{k_z} S_i z_{in'} \right) = 0. \end{aligned} \quad (20)$$

Consider first the ion-kinetic mode of the shear-flow-driven EIC instability. The approximate instability threshold at the short wavelength limit along the magnetic field can be found from Eq. (20). Assuming that the electrons are adiabatic we equate to zero the real and imaginary parts of the Eq. (20), and obtain the set of equations for variables λ and $z_{in'}$, which is similar to the corresponding set for the first cyclotron harmonic (4)

$$\begin{cases} n'\lambda/\sqrt{2} + z_{in'} - \lambda k_y \rho_{Ti} S_i z_{in'} = 0, \\ 1 + \tau + \psi(z_{\perp}) - \lambda k_y \rho_{Ti} S_i A_{n'}(b_i) = 0. \end{cases} \quad (21)$$

This set has a solution when the inequality $k_y \rho_{Ti} S_i > 0$ is met. For this case we obtain the short-wavelength threshold value $\lambda_{n's}$ for the excitation of the instability, at which normalized complex frequency $z_{n's}$ becomes real,

$$\lambda_{n's} = \frac{1}{k_y \rho_{Ti} S_i A_{n'}(b_i)} (1 + \tau + \psi(z_{\perp})), \quad (22)$$

$$z_{n's} = \frac{n'}{\sqrt{2} S_i k_y \rho_{Ti}} \left(1 + \frac{A_{n'}(b_i)}{1 - G_{n'} + \tau} \right), \quad (23)$$

where $G_{n'} = A_{n'}(b_i) - \psi(z_{\perp})$ and index $n's$ means the short-wavelength threshold of instability for n' -th cyclotron harmonic. The value $\delta\omega = \delta\omega_{0n'}$ at the instability threshold is

$$\delta\omega_{0n'} = \frac{n' \omega_{ci} A_{n'}(b_i)}{1 - G_{n'} + \tau}. \quad (24)$$

The solution of Eq. (20) at the vicinity of stability threshold can be obtained by use the same approach as for the first cyclotron harmonic. This yields the dispersive part and the growth rate for ion cyclotron instability,

$$\delta\omega \simeq \delta\omega_{0n'} \frac{\lambda_{n's}}{\lambda} + \delta\omega_{0n'} \left(\frac{\sqrt{2} k_y \rho_{Ti} S_i \text{Im}W(z_{n's})}{\sqrt{\pi} n' |W(z_{n's})|^2} - \frac{A_{n'}(b_i)}{1 - G_{n'} + \tau} \right) \left(1 - \frac{\lambda_{n's}}{\lambda} \right), \quad (25)$$

$$\gamma \simeq \delta\omega_{0n'} \frac{\sqrt{2} k_y \rho_{Ti} S_i \text{Re}W(z_{n's})}{\sqrt{\pi} n' |W(z_{n's})|^2} \left(1 - \frac{\lambda_{n's}}{\lambda} \right). \quad (26)$$

The growth rate is affected by threshold value $z_{n's} \approx n'/\sqrt{2} k_y \rho_{Ti} S_i$, because $\gamma \propto \text{Re}W(z_{n's}) = \exp(-z_{n's}^2)$. The growth rate of instability is exponentially small when the flow velocity shear and the transverse wave number are such that $\sqrt{2} k_y \rho_{Ti} S_i < n'$, whereas, at opposite inequality the growth rate is not exponentially small. The magnitude of the growth rate is also affected by the factor $k_y \rho_{Ti} A_{n'}(b_i)$. The function $A_{n'}(b_i)$ at $k_y \rho_{Ti} \sim n' \gg 1$ has asymptotic form

$$A_{n'}(b_i) \sim \left(1/\sqrt{2\pi} k_y \rho_{Ti} \right) \exp(-n'^2/2k_y^2 \rho_{Ti}^2), \quad (27)$$

which implies that long waves with $k_y \rho_{Ti} < n'$ have exponentially small growth rate. Thus, waves with longitudinal wavelength $\lambda > \lambda_{n's}$ and transverse wavenumbers such as $k_y \rho_{Ti} \gtrsim n'$ and $\sqrt{2} k_y \rho_{Ti} S_i > n'$

are unstable. We note also that the threshold wavelength $\lambda_{n's}$ (22) with these transverse wavenumbers is approximately equal to the corresponding magnitude for the first cyclotron harmonic (5) with transverse wavenumbers $k_y \rho_{Ti} \gtrsim 1$.

As it is for the fundamental ion cyclotron harmonic, the growth rate of the ion-kinetic mode of the shear-flow-driven EIC instability increases with an increase of wavelength along the magnetic field. Simultaneously the value $|z_{in'}|$ increases also. When the transverse wave numbers meet the inequality $\sqrt{2}k_y \rho_{Ti} S_i < n'$, the growth rate, remaining exponentially small quantity, reaches a maximum at a certain value of λ and then rapidly decreases. When $\sqrt{2}k_y \rho_{Ti} S_i > n'$, the ion-kinetic mode with the fulfilment of condition $|z_{in'}| > 1$ turn into the ion-hydrodynamic mode. The dispersion equation for the ion-hydrodynamic mode of the shear-flow-driven EIC instability can be obtained from Eq. (20) by using the asymptotic form of W - function for large argument values

$$1 + \tau \left(1 + i\sqrt{\pi} z_{e0} W(z_{e0}) \right) - G_{n'} - \frac{A_{n'}(b_i)}{\sqrt{2}z_{in'}} \lambda + \frac{A_{n'}(b_i)}{2z_{in'}^2} k_y \rho_{Ti} S_i \lambda = 0. \quad (28)$$

where $z_{e0} = n' \lambda / \sqrt{2\mu}$. The solution of Eq. (28) is

$$z_{in'} = \left[\lambda A_{n'}(b_i) \pm \sqrt{(\lambda A_{n'}(b_i))^2 - 4 \left(1 + \tau \left(1 + i\sqrt{\pi} z_{e0} W(z_{e0}) \right) - G_{n'} \right) S_i \lambda k_y \rho_{Ti} A_{n'}(b_i)} \right] \times \left[2\sqrt{2}k_y \rho_{Ti} \left(1 + \tau \left(1 + i\sqrt{\pi} z_{e0} W(z_{e0}) \right) - G_{n'} \right) \right]^{-1}. \quad (29)$$

The value z_{e0} may be more or less than the unity depending on the number of cyclotron harmonic. We find from Eq. (29) the long wavelength threshold for n' -th harmonic of the ion-hydrodynamic mode, assuming that $z_{e0} \gg 1$. The instability develops when the wavelength λ meets the inequality

$$\lambda < \lambda_{n'lh} = \frac{4k_y \rho_{Ti} S_i}{n'^2 A_{n'}(b_i)} (1 - G_{n'}), \quad (30)$$

where index $n'lh$ means the long-wavelength threshold for the n' -th cyclotron harmonic of the ion-hydrodynamic mode. For the transverse wavenumbers $k_y \rho_{Ti} \gtrsim n'$, the long-wavelength threshold (30) is equal approximately to the corresponding value for the first cyclotron harmonic (16) with transverse wavenumbers $k_y \rho_{Ti} \gtrsim 1$. The dispersion and the growth rate of EIC oscillations obtained from (29) are:

$$\delta\omega = \frac{\delta\omega_{0n'}}{2}, \quad \gamma = \frac{\delta\omega_{0n'}}{2} \sqrt{\frac{\lambda_{n'lh} - \lambda}{\lambda}} \quad (31)$$

which is equal approximately to the corresponding values, obtained above for the first cyclotron harmonic. The growth rate (31) far from the stability threshold, as well as growth rate (26) for the kinetic mode, varies slightly with changing $k_y \rho_{Ti}$ for $k_y \rho_{Ti} > n'$, while for opposite inequality it decreases exponentially with decreasing of $k_y \rho_{Ti}$. With decreasing of λ , the growth rate increases until the value $|z_{in'}|$ becomes of the order of unity, for which the asymptotic form of W - function, obtained for $|z_{in'}| > 1$ becomes unapplicable. As it was shown by numerical estimates, the growth

rate has a maximum at $|z_{in'}| \sim 1$, i. e. on the boundary of the ion-kinetic and ion-hydrodynamic modes.

When the inequality $\lambda > \lambda_{n'lh}$ holds, the growth rate of EIC ion-hydrodynamic mode is zero. In this case, the ion cyclotron oscillations split onto two hydrodynamically stable EIC modes, namely, current-driven modified by shear and shear-flow-driven modes. For these modes

$$\delta\omega = \frac{\delta\omega_{0n'}}{2} \left(1 \pm \sqrt{\frac{\lambda - \lambda_{n'l}}{\lambda}} \right). \quad (32)$$

For the excitation of the electron-kinetic shear-flow-driven EIC instability in plasma without electron current, the inequality $z_{e0} = n'\lambda/\sqrt{2\mu} < 1$ with $\lambda > \lambda_{n'lh}$ must hold. Hence, the restriction on the numbers of unstable cyclotron harmonics can be obtained, $n' < \sqrt{2\mu}/\lambda_{n'lh}$. The last inequality may be satisfied only for heavy ions; specifically, for the oxygen ions we have $n' < 6$.

Note, that the ion-hydrodynamic mode does not excite for values of the flow velocity shear and transverse wave numbers, for which $\sqrt{2}k_y\rho_{Ti}S_i < n'$; the splitting of EIC oscillations on two modes also is absent in that case. Thus, a necessary condition for the excitation of high n' -th harmonics of shear-flow-driven EIC instability is determined by the inequalities $\sqrt{2}k_y\rho_{Ti}S_i > n'$ and $k_y\rho_{Ti} \gtrsim n'$ which are similar to the corresponding conditions for the fundamental cyclotron harmonic.

4 CONCLUSIONS

In this paper, we have studied the EIC instabilities driven by the velocity shear of plasma flow along the magnetic field (the shear-flow-driven EIC instability) for different values of the wavelength along the magnetic field. The EIC oscillations with first $n = 1$ and high $|n| \geq 2$ cyclotron harmonics are considered. It was shown that the shear-flow-driven EIC instabilities, depending on the wavelengths along the magnetic field, are of three types: the ion-kinetic mode, the ion-hydrodynamic mode and the electron-kinetic mode. Each of these modes are caused by different mechanisms.

The ion-kinetic mode is the most short-wavelength among them. It develops as a result of the combined effect of the inverse of ion cyclotron damping and leads to the ions heating along the magnetic field. The short-wavelength limit along the magnetic field λ_{ns} for the ion-kinetic shear-flow-driven EIC instability for $n = 1$ and $|n| \geq 2$ cyclotron harmonics were determined by Eqs. (5) and (22), respectively, which are almost equal. The ion-hydrodynamic mode is dominant for longer wavelength of ion cyclotron waves along the magnetic field. The longitudinal wavelength λ_{bn} , which conventionally separates these modes is determined by equality $|z_{in}| = \lambda_{bn}|\omega - n\omega_{ci} - k_z V_{0i}|/\sqrt{2}\omega_{ci} \simeq 1$ and the magnitudes λ_{bn} for $n = 1$ and $|n| \geq 2$ cyclotron harmonics are approximately equal. Thus, the ion-kinetic mode is excited at wavelengths $\lambda_{ns} < \lambda \lesssim \lambda_{bn}$. We have numerically obtain that the growth rate of the shear-flow-driven EIC instability has a maximal value at the boundary of ion-kinetic and ion-hydrodynamic modes, i. e. at $\lambda \simeq \lambda_{bn}$.

The ion-hydrodynamic shear-flow-driven EIC instability has the similar mechanism of development, as for the aperiodic instability driven by velocity shear, which was discovered by D'Angelo[17]. It exists in the range of wavelengths along the magnetic field, determined by inequality $|z_{in}| > 1$. The dispersion as well as the growth rate of the shear-flow-driven EIC instability are approximately the same for all cyclotron harmonics. The ion-hydrodynamic mode has a limited value of the long wavelength along the magnetic field λ_{nlh} , which is determined by Eqs. (15) and (30) for $n = 1$ and $|n| \geq 2$ cyclotron harmonics respectively; in so doing for the transverse wave numbers $k_y \rho_{Ti} \gtrsim n$ the boundary wavelengths λ_{nlh} for different cyclotron harmonics are approximately equal. Thus the ion-hydrodynamic mode is excited in wavelengths range $\lambda_{bn} \lesssim \lambda < \lambda_{nlh}$. This mode leads to the turbulent heating of ions across the magnetic field due to the effect of ion cyclotron resonances broadening.

For the wavelengths $\lambda > \lambda_{nlh}$, the EIC oscillations split into two hydrodynamically stable modes with the dispersion evaluated by Eqs. (17) and (32) for $n = 1$ and $|n| \geq 2$ cyclotron harmonics respectively. One of them with sign minus in Eqs. (17) and (32) is the electron-kinetic mode of the shear-flow-driven EIC instability. It becomes unstable even in currentless plasma flow due to an inverse of electron Landau damping when $z_{e0} = n\lambda/\sqrt{2\mu} \lesssim 1$. The excitation of electron-kinetic mode depends on the ions mass and of cyclotron harmonic mode number. We have found that it may be excited only for heavy ions, such as oxygen ions, while for a hydrogen ions this mode is not excited. Then the range of wavelength of the electron-kinetic mode is evaluated by inequality $\lambda_{nhl} < \lambda \lesssim \lambda_{nl} \simeq \sqrt{2\mu}/n$.

We have obtained the conditions for the excitation of the shear-flow-driven EIC instabilities, which connects the normalized shear and wave numbers transversely the magnetic field. These conditions are identical for all considered modes, and determined as $\sqrt{2}k_y \rho_{Ti} S_i > n$. When this condition holds, all modes excited simultaneously. These instabilities result in the ion heating both along and across the magnetic field as well as in the electron heating along the magnetic field.

References

- [1] E.V.Belova, Ya. Blensky, M.Denis, L.M.Zelenyy, S.P.Savin, Sov. Plasma Physics, **17**, 401 (1991).
- [2] V.V.Gavrishchaka, G.I.Ganguli, W.A.Scales, S.P.Slinker, C.C.Chaston, J.P.McFadden, R.E.Ergun, and C.W.Carlson, Phys. Rev. Lett. **85**, 4285 (2000).
- [3] G.Ganguli, S.Slinker, V.Gavrishchaka, and W.Scales, Phys. Plasmas **9**, 2321 (2002).
- [4] E.E.Scime, R.Murphy, G.Ganguli, and E.Edlund, Phys. Plasmas **10**, 4609 (2003).
- [5] R.L.Merlino, Phys. Plasmas **9**, 1824 (2002).
- [6] V.S.Mikhailenko, D.V.Chibisov, and V.V.Mikhailenko, Phys. Plasmas **13**, 102105 (2006).

- [7] V.S.Mikhailenko and D.V.Chibisov, Phys. Plasmas **14**, 082109 (2007).
- [8] V.S.Mikhailenko, V.V.Mikhailenko and K.N.Stepanov, Phys. Plasmas **15**, 092901 (2008).
- [9] C. Teodorescu, E.W.Reynolds, and M.E.Koepke, Phys. Rev. Lett. **89**, 105001 (2002).
- [10] S-H. Kim, E.P.Agrimson, M.J.Miller, N. D'Angelo, R.L.Merlino and G.I.Ganguli, Phys. Plasmas **11**, 4501 (2004).
- [11] T.Kaneko, H.Saito, S.Ishiguro, R.Hatakeyama, J. Plasma Phys. **72**, 989 (2006).
- [12] W.E.Amatucci, J. Geophys. Res. **104**, 14481 (1999).
- [13] M.E.Koepke, C. Teodorescu and E.W. Reynolds, Plasma Phys. Control. Fusion **45**, 869 (2003)
- [14] P. Norqvist, M. André, L. Eliasson, A. I. Eriksson, L. Blomberg, H. Lühr, and J. H. Clemmons, J. Geophys. Res., **101**, 13179 (1996)
- [15] C.A.Cattell, F.S. Mozer, I.Roth, R. R. Anderson, R.C.Elphic, W.Lennartsson, E.Ungstrup, J.Geophys. Res., **96**, 11421 (1991)
- [16] C.Cattell, R.Bergmann, K.Sigsbee, C.Carlson, C. Chaston, R.Ergun, J.McFadden, F.S. Mozer, M.Temerin, R. Strangeway, R. Elphic, L. Kistler, E. Moebius, L. Tang, D. Klumpar, R. Pfaff, Geophys. Res. Lett., **25**, 2053 (1998)
- [17] N. D'Angelo, Phys. Fluids, **8**, 1748 (1965)